Nonlinear AVO inversion using geostatistical a priori information

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Abstract

We present a Monte Carlo based strategy for non-linear inversion of seismic amplitude versus offset data. The problem is formulated in a Bayesian framework such that the solution to inverse problem is an a posteriori probability density. A priori information about the problem is defined as a Gaussian probability density. The problem is conditioned by observations of reflected P-waveforms. A nonlinear relation between the model and data based on ray tracing and Zoeppritz equations is considered. As a consequence of these nonlinearities, no closed form expression of the a posteriori probability density can be formulated. Therefore, we apply the Metropolis algorithm in order to obtain an exhaustive characterisation of the a posteriori probability density.

In geophysical inverse problems the model is often discretized into a huge number of model parameters, which results in a high dimensional a posteriori probability density to be sampled. Traditional applications of the Metropolis Algorithm involves that a single model parameter is perturbed according to the a priori information. Each perturbation requests an evaluation of the Metropolis rule, which includes an evaluation of the (often) computationally expensive forward relation. Geophysical model parameters, like subsurface elastic parameters, are often defined in Euclidean space. Therefore, we suggest using geostatistical algorithms, which are designed to handle spatial distributed data set, for defining a priori information about the inverse problems. We demonstrate how the geostatistical Fast Fourier Transform Moving Average algorithm provides a means of multi-parameter perturbations of the a priori information. In this way an efficient sampling using the Metropolis algorithm is obtained. The suggested strategy is tested on a synthetic amplitude versus offset data set.

Keywords: Nonlinear inversion, Metropolis Algorithm, Geostatistics, A priori information, Amplitude Versus Offset

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1. INTRODUCTION

Inversion of seismic amplitude versus offset (AVO) data provides information about the subsurface elastic parameters. Reflection seismic prestack recordings contain information about the lithology, which is crucial for the inquiry of hydrocarbon reservoirs that are related to lithological heterogeneities rather than geological structures. Reliable estimates of the elastic parameters are important in order to obtain trustworthy information about the lithology.
In this study we use a Bayesian formulation of the solution to the inverse problem. In this way not only a single estimate but multiple realizations are used to characterize the solution of the inverse problem. This, in turn, provides a means of resolution analysis of the solution (Mosegaard, 1998).

Works of e.g. Buland and Omre (2003) and Kjønsberg et al. (2010) suggest using a Bayesian approach for AVO inversion in order to obtain samples of the a posteriori probability density. In their approaches a weak contract approximation of Zoeppritz equation is used as the forward relation of the problem, which may lead to approximations of the solution in case of high contrast lithological variations.

In this study the forward relation (that relates the elastic parameters to the seismogram) is calculated using ray-tracing and Zoeppritz equation for the P- to P-wave reflectivity coefficients. Reflectivity models obtained in this way are subsequently convolved with the source wavelet in order to obtain the resulting seismogram. Using this forward relation, no approximate pre-inversion angle-dependent data sorting is needed and, instead, common midpoint data can be used directly. Moreover, this fully nonlinear formulation of the problem allows for inversion of the parameters at their actual depth coordinates rather than positions given as two way travel times.

A Monte Carlo based inversion strategy for inversion of AVO data is suggested. This approach combines the Metropolis algorithm with the Fast Fourier Transform Moving Average (FFT-MA) algorithm (Le Ravalec, 2000) and the Gradual Deformation Method (GDM) (Hu, 2000). We suggest using geostatistical algorithms in order to describe and sample a priori information of geophysical inverse problems since these algorithms are designed to model spatial distributed data (Hansen et al., 2008). In this paper we chose to use the FFT-MA algorithm because this algorithm is very efficient at generating Gaussian random fields, even for very large scales. A combination of the FFT-MA algorithm with the GDM algorithm seems to provide a flexible tool for controlling the perturbation step sizes when applied as a priori sampler in conjunction with the Metropolis algorithm (Mosegaard and Tarantola, 1995).

The suggested algorithm is used to sample the a posteriori probability of a 1D elastic model conditioned by synthetic AVO data.

2. METHODOLOGY

The subsurface is discretized into an N layer model. Each layer is represented by a P-wave velocity ($\mathbf{v}_p$), an S-wave velocity ($\mathbf{v}_s$), and a mass density ($\boldsymbol{\rho}$). Thus, the subsurface can be represented by 3xN model parameters, $\mathbf{m}$ (i.e. the model). Data are given by a CMP gather $\mathbf{s}$ of recorded P-wave seismograms. The forward problem that relates a model with the CMP gather can be represented by the following three equations (Sheriff and Geldart, 1982; Yilmaz, 1987):

$$\boldsymbol{\alpha}(z,t) = g_{raytracing}\left(\mathbf{v}_p(z)\right), \tag{1}$$

$$\mathbf{r}_{pp}(z) = g_{zoeppritz}\left(\boldsymbol{\alpha}(\mathbf{v}_p(z)), \mathbf{v}_p(z), \mathbf{v}_s(z), \boldsymbol{\rho}(z)\right), \tag{2}$$

$$\mathbf{s}(t) = \mathbf{W}\mathbf{r}_{pp}(t), \tag{3}$$

where eq. (1) is solved through a ray-tracing algorithm in the P-wave velocity-depth model in order to determine the two way travel times t and reflection angles α at the N-1 interfaces in the N layered model. The associated P- to P-wave reflection coefficients $\mathbf{r}_{pp}$ at the individual interfaces are obtained through Zoeppritz equations (eq. (2)). The final CMP gather is calculated using the convolution model which relates the source wavelet, $\mathbf{W}$, and the reflectivity model, $\mathbf{r}_{pp}(t)$, with the seismogram, $\mathbf{s}(t)$. Notice that the reflectivity model can be transformed from a function of depth into a function of time since the ray-tracing provides the two way travel times to the individual layers. In practice this step involves an interpolation to an equidistant temporal sampling interval as the travel times and depths are nonlinearly related through eq. (1). Eq. (2) is also a nonlinear relation whereas eq. (3) is linear. Hence, the resulting problem of inferring the elastic parameters as a function of depth is a nonlinear inverse problem.

In a Bayesian formulation the solution to the inverse problem is given as an a posteriori probability density of the model which can be formulated as (e.g. Tarantola, 2005):

$$\sigma_M(\mathbf{m}) = k\rho_M(\mathbf{m})L(\mathbf{d},\mathbf{m}), \tag{4}$$

where k is a normalization constant, $\rho_M(\mathbf{m})$ is the a priori probability density, and $L(\mathbf{d},\mathbf{m})$ is the likelihood function. $\rho_M(\mathbf{m})$ describes the probability that the model satisfies the a priori information. $L(\mathbf{d},\mathbf{m})$ describes how well the modelled data explains the observed data given a data uncertainty. In this study both the a priori probability density and the likelihood function are characterized as Gaussian probability densities:

$$\rho_M(\mathbf{m}) \sim N(\mathbf{m}_0, C_m) \tag{5}$$

$$L(\mathbf{d},\mathbf{m}) \sim N(\mathbf{d}_{obs}, C_d) \tag{6}$$

The a posteriori probability density is a non-Gaussian distribution because the forward relation of the AVO responses is nonlinearly related to the elastic parameters of the subsurface. As a consequence no closed form expression can be obtained for the a posteriori probability density. Therefore, the solution to the inverse problem has to be characterized through sampling the a posteriori probability density. One way of obtaining samples from a high dimensional probability density is by using the Metropolis algorithm. The minimum requirement of the algorithm is; 1) a “black box” algorithm that is able to produce samples of the a priori probability density and, 2) a “black box” algorithm that is able to evaluate the likelihood function for a given model. The flowchart of the Metropolis algorithm is a follows (Mosegaard and Tarantola, 1995):

1) An a priori sampler proposes a sample, $\mathbf{m}_{propose}$, from the a priori probability density, which is a perturbation of a previously accepted model, $\mathbf{m}_{accept}$.

2) The proposed sample is accepted with the probability (known as the Metropolis rule):

$$P_{accept} = \min\left(1, \frac{L(\mathbf{m}_{propose})}{L(\mathbf{m}_{accept})}\right) \tag{7}$$

3) If the proposed model is accepted, $\mathbf{m}_{propose}$ becomes $\mathbf{m}_{accept}$. Otherwise the $\mathbf{m}_{propose}$ is rejected and $\mathbf{m}_{accept}$ remains.

4) The procedure is continued until a desirable number of models have been accepted. All models accepted by the Metropolis algorithm (counted with possible repetitions) are samples of the a posteriori probability density.

The Fast Fourier Moving Average (FFT-MA) generator is an efficient way to obtain samples from a Gaussian probability density (Le Ravalec, 2000). In this study the FFT-MA algorithm will serve as the “black box” algorithm that provides samples of a Gaussian a priori probability density. The Moving Average strategy uses, unlike a Cholesky decomposition approach, a covariance function instead of a covariance matrix. This approach is considerably superior since realizations of a very large field can be obtained in very short time. The covariance function can be written as a convolution product of a function *g* and its transpose (e.g. Journel and Huijbregts, 1978):

$$c = g * \breve{g} \tag{8}$$

If *g* can be obtained, a Gaussian random field with mean *m* and covariance *c* is obtained as (e.g. Journel and Huijbregts, 1978):

$$y = m + g * z\,, \tag{9}$$

where *z* is a field of Gaussian white noise. The Gaussian random field is in practice obtained using a Fast Fourier Transform (FFT) algorithm to transform *g* into the Fourier domain.

In order to obtain an efficient Metropolis algorithm it is necessary to be able to control the perturbation steps of the a priori probability density. The Gradual Deformation Method (Hu, 2000) is adequate for controlling the degree of perturbation from one Gaussian realization to another. Consider two independent Gaussian fields z_1 and z_2. The GDM algorithm can be used to obtain a new Gaussian field, z, that is a linear combination between two independent Gaussian fields (Hu, 2000):

$$z = z_1 \cos(\pi p) + z_2 \sin(\pi p)\,, \tag{10}$$

where the parameter p takes on values between 0 and ½. Since the FFT-MA algorithm relies on Gaussian white noise it does not matter whether the entire field or only a subarea of z is perturbed. Eq. (10) can be rephrased as:

$$z_{propose} = z_{current} \cos(\pi p) + z_{new} \sin(\pi p)\,, \tag{11}$$

where z_{new} is either a completely new realization of Gaussian white noise or a new realization of a subarea of the current field $z_{current}$.

Eq. (11) can then be used to generate perturbations of a Gaussian a priori probability density using the following steps (Le Ravalec, 2000):
1) An initial unconditional sample of the a priori probability density is calculated using eq. (9) (i.e. FFT-MA algorithm) based on an initial distribution of normal derivatives, $z_{current}$.
2) A "subarea" of $z_{current}$ is randomly chosen and a new field of Gaussian white noise is substituted with the values within the subarea resulting in a new distribution of Gaussian white noise denoted z_{new}.
3) A new field of Gaussian white noise, $z_{propose}$, based on $z_{current}$ and z_{new} is calculated using eq. (10) (i.e. the GDM algorithm).
4) A new Gaussian field $y_{propose}$ with covariance *c* and mean *m* based on $z_{propose}$ is calculated using eq. (9).
5) By setting $z_{current}$ equal to $z_{propose}$ and repeating step 2 to 4, a desirable number samples from a Gaussian a prior probability density can be obtained.

The size of the subarea in step (2) and the value of the p-parameter govern the exploratory nature of the Metropolis algorithm. For relatively large subareas the perturbation step becomes relatively large. Likewise, relatively large p-values results in large perturbation steps. The size of the perturbation area and the p-value have to chosen subjectively. For a subarea extension larger than one model parameter we suggest using a p-parameter value of ½ and regulate the size of the subarea in order to control the perturbation step. In case the subarea constitutes only a single model parameter the adjustment of the p-value should be used as the regulating parameter.

One strategy for choosing these parameters is to adoptively adjust the size of the subarea or the p-value during the sampling such that a certain acceptance probability of the Metropolis rule is maintained. Gelman et al. (1996) found that the acceptance rate should be around 23% for high-dimensional distributions. For large acceptance rates the algorithm is exploring the a posteriori probability density too slowly. On the other hand, for smaller acceptance rates too many computationally expensive trials are performed. A constant acceptance rate results in a larger perturbation step in the initial (burn-in) period of the sampling than in the subsequent sampling period. In this way the algorithm becomes efficient at detecting areas in the model space of significant probability in the burn-in phase and then adopts to smaller exploration steps. This approach, however, implicitly assumes a constant global smoothness of the a posteriori probability density, which may result in unexplored areas of the distribution. On the other hand, however, this approach is expected to be appropriate for moderately nonlinear (i.e. nonGaussian) a posteriori probability densities.

Finally, the likelihood function is defined as:

$$L(s) = k \exp\left(-\frac{1}{2} \sum_{i=1}^{N} \left(s^i - s^i_{obs} \right) / \sigma^2 \right), \qquad (12)$$

Where s^i represents the amplitude of the simulated seismic waveforms obtained using eqs. (1) – (3). s^i_{obs} are the sample points of the observed AVO data. σ is the standard deviation of the expected data uncertainty.

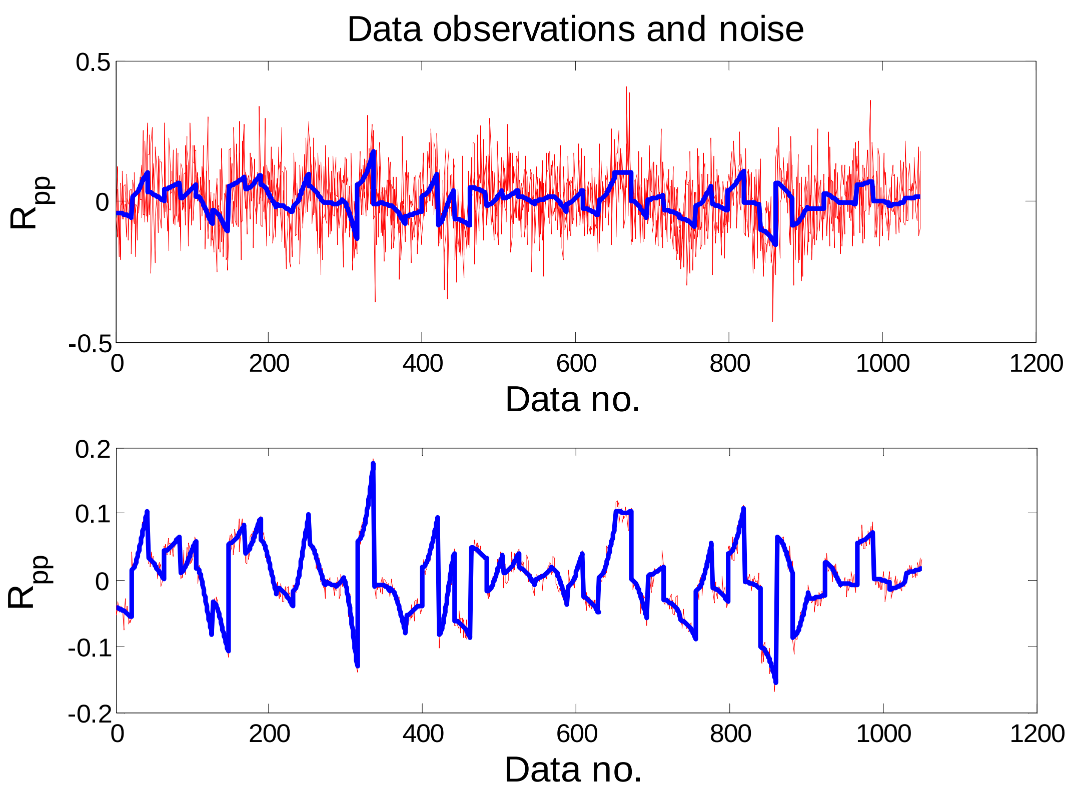

Fig. 1: The synthetic data are plotted as the blue curve. The red curve represents the sum of the synthetic data and the noise component. In the topmost figure a zero mean Gaussian noise component with a standard deviation of 0.1 has been added to the data. In the lower most figure a similar noise component with a standard deviation of 0.01 has been added to data. The resulting signal-to-noise (S/N) ratios of topmost and lowermost signals are 0.24 and 24, respectively.

3. RESULTS AND DISCUSSION

A synthetic reference model is calculated using eq. (9) (i.e. the FFT-MA algorithm). The reference model is a 1000 m deep 50 layer 1D model. The values of the elastic parameters of each layer are seen as the red curve in Figs. 2 and 4. The signals are recorded with a receiver spacing of 100 m and a maximum offset of 2000 m. Ray-tracing and the associated reflection angels are calculated using the Matlab function *traceray_pp* from the CREWES software package (Margrave, 2003). P- to P-wave reflection coefficients can be calculated using Zoeppritz equations. In a field example the observed data are recorded P-waveforms. Thus, a source pulse has to be convolved with the reflection coefficients in order to obtain the resulting seismogram. In the present synthetic example we will not perform the convolution and the synthetic observed data are given as reflection coefficients. Zero mean Gaussian white noise with a standard deviation of 0.1 and 0.01, respectively, is added to the reflection coefficients (see Fig. 1). A correct assumption about the noise is accounted for in Likelihood function. The results of using the suggested sampling strategy are seen in Figs. 2 – 5. Blue curves show statistically independent realizations of the a posteriori probability density. Green curves are the mean of the (blue) realizations. The dotted black curves in Figs. 2 and 4 show the 0.95 confidence interval of the a priori probability density. Hence, deviations from the a priori distribution can be ascribed to information provided by data.

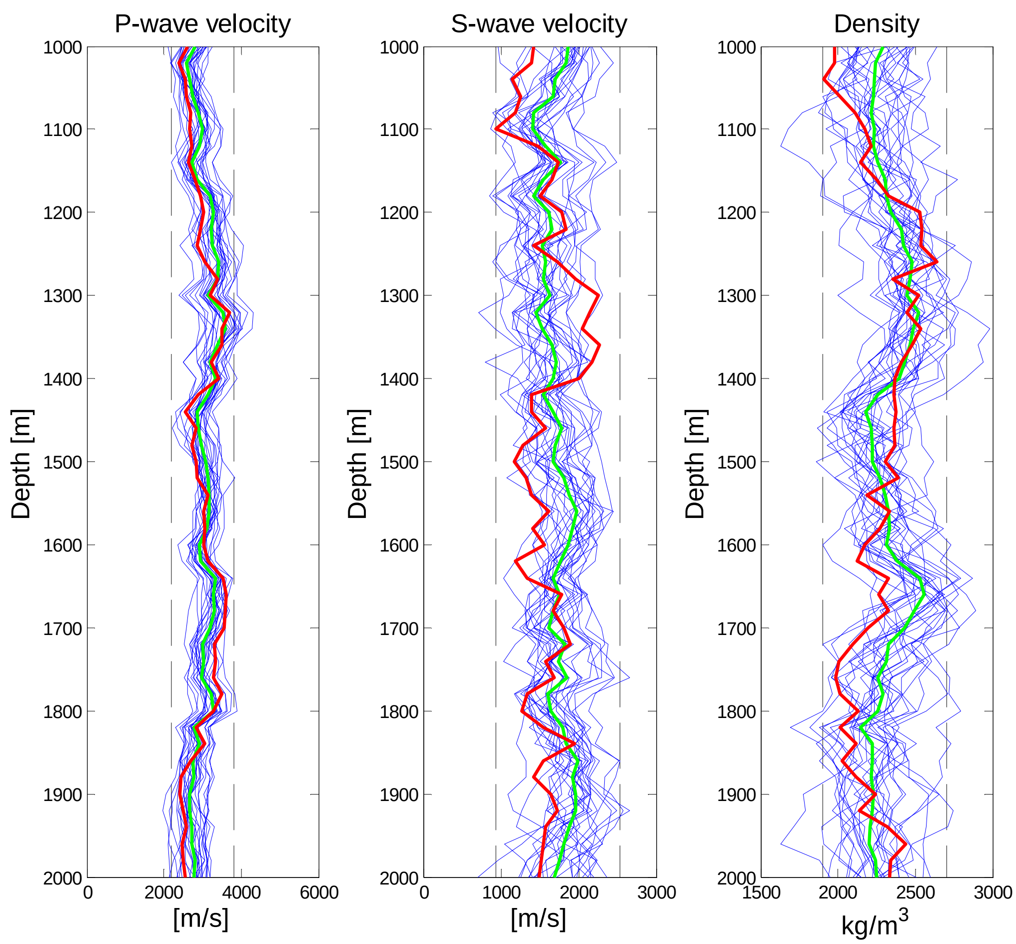

Fig. 2: Results of the inversion using a zero mean normal distributed noise component with a standard deviation of 10^{-1} added to the reflectivity coefficients. The red curve is the “true” reference model, the blue curves are realizations of the a posteriori probability distribution, and the green curve is the mean of the (blue) realizations. The dotted black lines show the 0.95 interval of the a priori probability distribution.

The results show that even though a strong noise component has been added to the data the a posteriori distribution still provides considerable more information about the model parameters than the prior information. In particular, information about the $\mathbf{v}_p$ is resolved. Some information about the density distribution $\boldsymbol{\rho}$ is obtained, whereas least information of the $\mathbf{v}_s$ structure is obtained (see Fig. 2). Fig. 3 shows the result of combining the a posteriori realizations of $\mathbf{v}_p$, $\mathbf{v}_s$, and $\boldsymbol{\rho}$ into $\mathbf{v}_p/\mathbf{v}_s$ ratio, acoustic impedance, and shear impedance. This result reveals that the high spatial frequencies of the $\mathbf{v}_p/\mathbf{v}_s$ ratio are recovered and that the entire spatial frequency spectrum of the acoustic impedance has been resolved. However, a combination of the two least resolved parameters ($\mathbf{v}_s$ and $\boldsymbol{\rho}$) into shear impedance only provides negligible information.

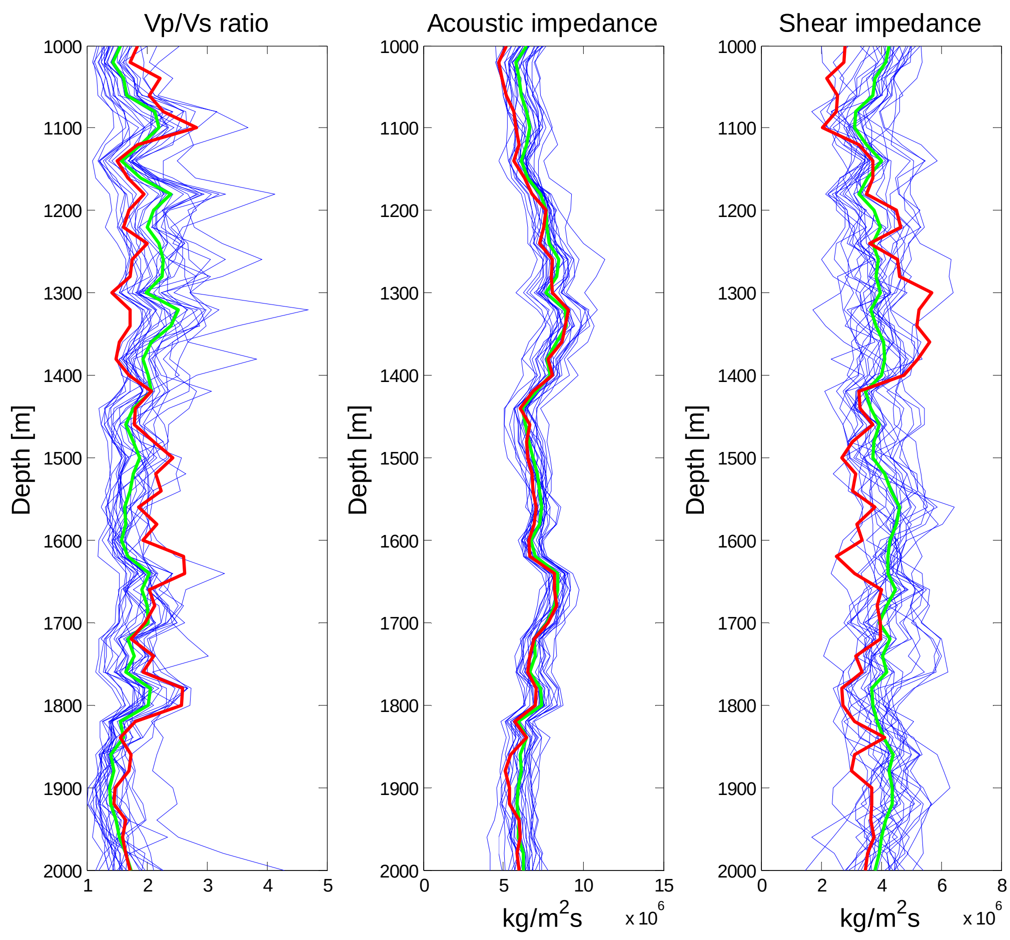

Fig. 3: Results of the inversion using a zero mean normal distributed noise component with a standard deviation of 10^{-1} added to the reflectivity coefficients. The red curve is the “true” reference model, the blue curves are realizations of the a posteriori probability distribution, and the green curve is the mean of the (blue) realizations. The dotted black lines show the 0.95 interval of the a priori probability distribution.

The standard deviation of the noise component is reduced to 0.01. The results of the inversion are depicted in Figs. 4 and 5. The results demonstrate that data are now able to resolve information about both the P- and S-wave velocity as well as the density. However, data seems to provide more information about the high spatial frequencies than compared to the low frequencies. Again it is seen that realizations of the $\mathbf{v}_p/\mathbf{v}_s$ ratio, acoustic impedance, and shear impedance yields an improved resolution as compared to the elastic parameters themselves.

Geostatistical algorithms are often designed for both 2D or 3D application, which is also the case for the FFT-MA algorithm. Hence, a 2D or 3D extension of the suggested inversion strategy can easily be obtained. The computational expensive part of such an extension would be related to a 3D ray tracing.

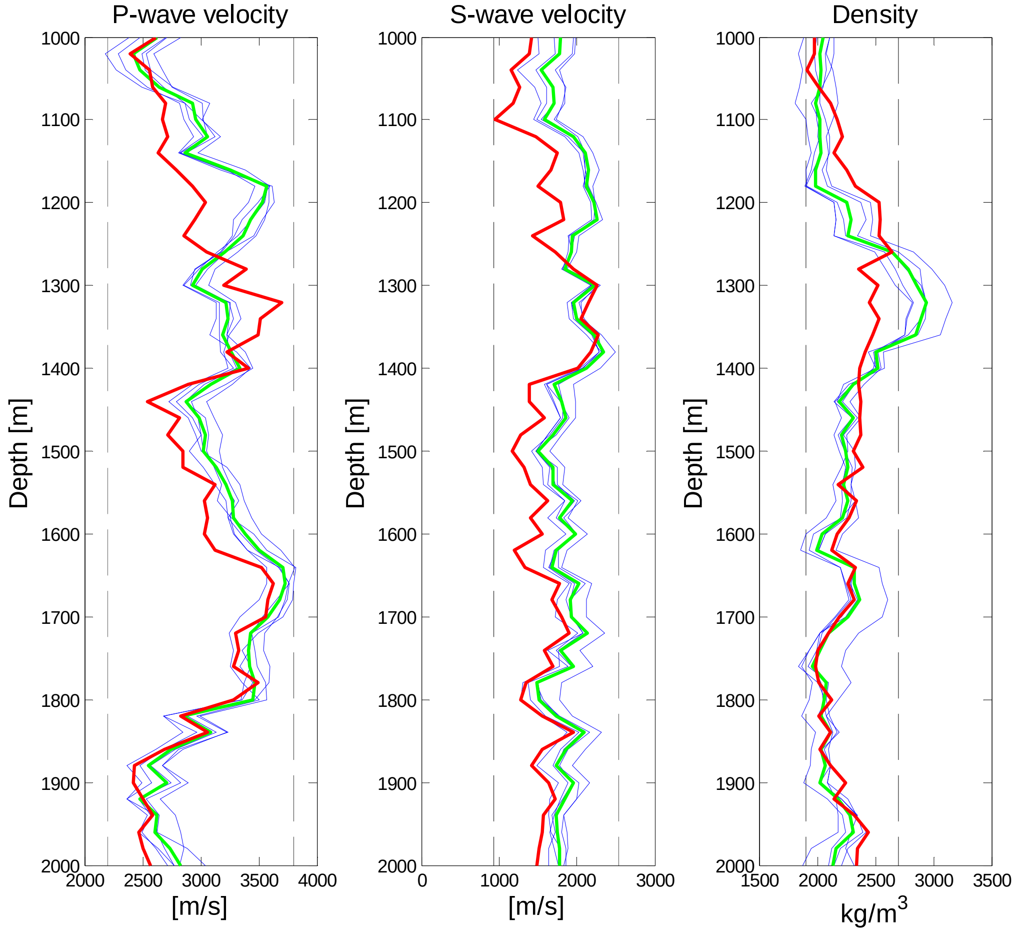

Fig. 4: Results of the inversion using a zero mean normal distributed noise component with a standard deviation of 10^{-2} added to the reflectivity coefficients. The red curve is the “true” reference model, the blue curves are realizations of the a posteriori probability distribution, and the green curve is the mean of the (blue) realizations. The dotted black lines show the 0.95 interval of the a priori probability distribution.

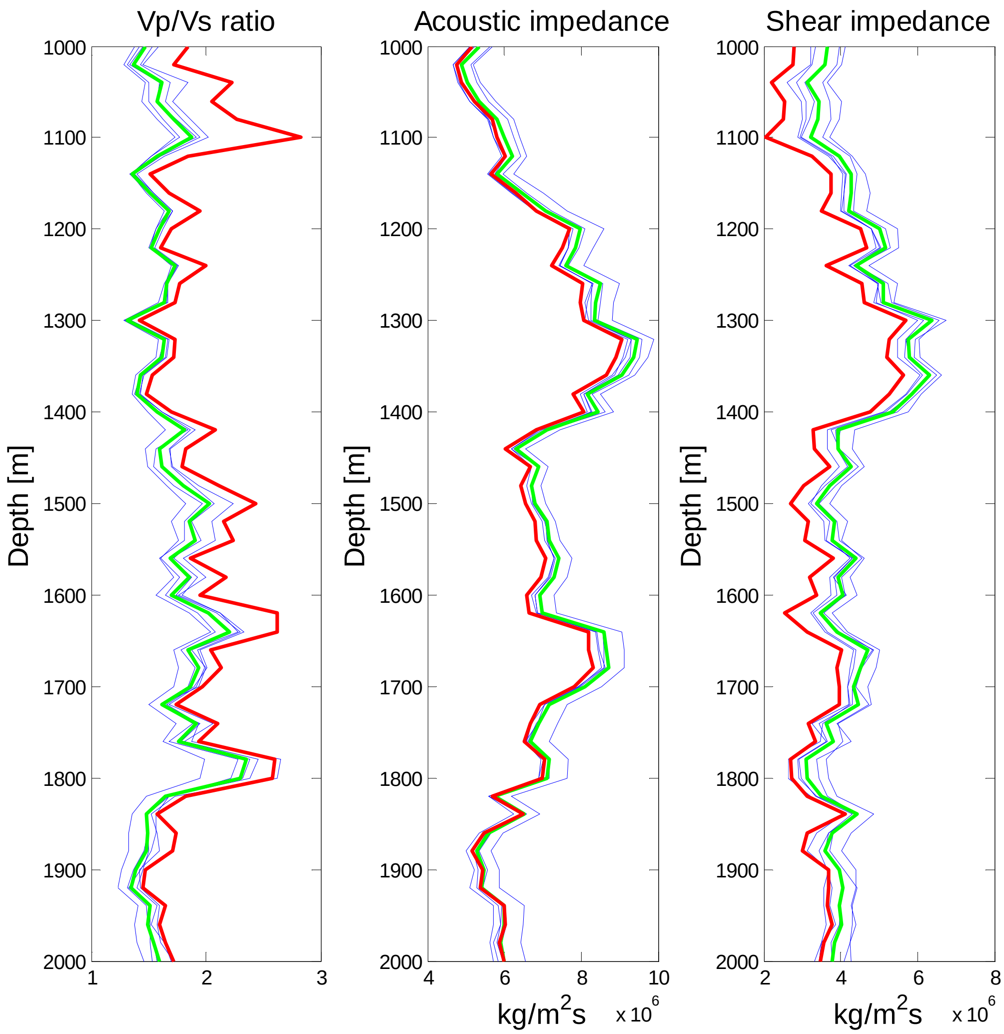

Fig. 5: Results of the inversion using a zero mean normal distributed noise component with a standard deviation of 10^{-2} added to the reflectivity coefficients. The red curve is the "true" reference model, the blue curves are realizations of the a posteriori probability distribution, and the green curve is the mean of the (blue) realizations. The dotted black lines show the 0.95 interval of the a priori probability distribution.

4. CONCLUSION

A nonlinear seismic AVO inverse problem has been formulated using a Bayesian framework. Hence, the solution to the inverse problems is an a posteriori probability density. A strategy combining the FFT-MA algorithm and the Metropolis algorithm has been suggested for sampling of the a posteriori probability density. The strategy has been tested on synthetic data. Even for a S/N ratio of as low as 0.24 the data provide reasonable information about elastic parameters. In particular, a good resolution of the acoustic impedance was obtained. Increasing the S/N ratio to 24 resulted in a considerable improvement in the resolution of all the elastic parameters. Regardless of the S/N ratio it was observed that the AVO data provided more information about the combined elastic parameters than for the elastic parameters themselves. In particular the results provided a reliable resolution of the acoustic impedance.

5. ACKNOWLEDGEMENT

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